

Calculus Course Objectives and Competencies:

The objectives in Calculus include ...

1. to develop skills and work problems involving functions and models (Calculus I).
2. to develop skills and work problems involving limits and rates of change (Calculus I).
3. to develop skills and work problems involving derivatives (Calculus I).
4. to develop skills and work problems involving applications of derivatives (Calculus I).
5. to develop skills and work problems involving anti-differentiation and integrals (Calculus I).
6. to develop skills and work problems involving applications of integration (Calculus I).
7. to develop skills and work problems involving inverse functions (Calculus II).
8. to develop skills and work problems involving techniques of integration (Calculus II).
9. to develop skills and work problems involving (further) applications of integration (Calculus II).
10. *to develop skills and work problems involving parametric equations and polar coordinates (Calculus II).*
11. *to develop skills and work problems involving conic sections and analytic geometry (Calculus II).*
12. *to develop skills and work problems involving infinite sequences and series (Calculus II).*

Note: Items 10-12 above will be added or omitted as time allows.

Upon successful completion of this course, the student should be able to:

1. *State a definition of real-valued function and use alternative methods by which such functions may be pictured or represented.*
2. *Define absolute value and solve elementary equations or inequalities involving absolute values.*
3. *Given an appropriate function f , identify the independent and dependent variables; find the domain and range of f ; calculate the value of f at a specified point; graph f by making a table of pairs.*
4. *Describe what each of the terms linear function, polynomial function, power function, exponential function, rational function, algebraic function and transcendental function mean, and given a specific function, identify which, if any, of these it is.*
5. *Given any two points on a straight line, or one point on the line and the slope of the line, find the equation of that line.*
6. *Find the equation for a circle, given its center and radius. Given an equation for a circle, find the coordinates of its center and the radius, and sketch its graph.*
7. *Graph any parabola of the form $ax^2 + bx + c$.*
8. *State the definition of a radian, convert between radian and degree measure, and, given an angle in radians, find its sine, cosine, tangent, cotangent, secant, and cosecant.*
9. *Graph the sine, cosine, tangent, cotangent, secant, and cosecant functions.*
10. *State and apply the trigonometric identities for the tangent, cotangent, secant, and cosecant in terms of the sine and cosine; for the sine and cosine of the sum and difference*

of two angles; between the squares of the sine and cosine of an angle; and between the squares of the tangent and secant of an angle.

11. *Given an equation for a function, write the equation for an appropriate shifted and/or stretched version of that same function. Graphically display the effect of the shifting and/or stretching on the original functions graph.*
12. *Given two functions f and g , find the sum, difference, product, quotient and composite.*
13. *State and apply the laws of exponents.*
14. *Graph exponential functions, i.e. $y = a^x$, for different appropriate values of a .*
15. *State what is meant by a one-to-one function, and given an appropriate function, determine whether or not it is one-to-one.*
16. *Describe what is meant by the inverse of a function. Given an appropriate function, determine its inverse either as an equation or graphically.*
17. *State the relationship between exponential notation and logarithmic notation. Use this to simplify logarithms of exponentials and exponentials of logarithms, and to solve appropriate exponential and logarithmic equations.*
18. *Use the properties of the natural logarithm to rewrite an appropriate logarithmic expression as a sum, difference, or multiple of logarithms.*
19. *State the relationship between the trigonometric and inverse trigonometric functions. Use this to simplify expressions involving trigonometric or inverse trigonometric functions.*
20. *Find the values of inverse trigonometric functions at selected points without the use of tables or a calculator.*

Note: Items 1-20 are considered prerequisite topics, and are not specifically covered, but are built upon.

21. Describe the differences between the average and instantaneous rates of change of a function, and, for a given function, interpret both of these graphically.
22. Describe briefly what is meant by the term limit, and by the symbol and notation "lim."
23. Know the general methods by which limits may be determined, and use these to determine limits for appropriate problems.
24. For an appropriate function $f(x)$, find the right hand and left hand limits as x approaches a , if they exist, and from these determine whether or not the "lim" of $f(x)$ exists (also issues of continuity).
25. State what the term vertical asymptote means, and for an appropriate function $f(x)$, find all of its vertical asymptotes.
26. Evaluate limits of expressions involving trigonometric functions by making use of appropriate trigonometric identities and the limit theorems.
27. State the requirements for a function $f(x)$ to be continuous at the point $x = a$. Given an appropriate function, determine its points of continuity and discontinuity, and briefly justify your selections.
28. Have a general understanding of the main theorems related to continuous functions as given in the text.
29. Describe and interpret graphically what is meant by "the tangent to a curve" at a given point.
30. State the definition of the derivative of the function f at the point $x = a$ and the relationships between the derivative, the tangent to the curve for f , and the instantaneous and average rate(s) of change of the function at that point.

31. State the definition of the derivative of a function, use the various equivalent different symbols used to represent it, and the relationships between the derivative function, the tangents to the curve for f at various points, the regions where f is increasing and decreasing, and the continuity of the original function.
 32. Given an appropriate function f , determine where it is differentiable, and find the derivative function at these points.
 33. State and apply the general rules for the derivative of a constant, a constant integer power, a constant multiple of a known function, and the sum and difference of two known functions.
 34. Find the derivative of any polynomial.
 35. State and apply the general rules for the derivative of products, quotients and powers of known functions.
 36. State and interpret graphically the derivatives of the sine and cosine. Find the derivatives of other trigonometric functions, making use of appropriate trigonometric identities and rules of differentiation.
 37. State and apply the chain rule to calculate derivatives of appropriate functions.
 38. Describe and apply the technique of implicit differentiation to compute the derivatives of appropriate functions.
 39. State the definition of the n^{th} derivative of a function, and given physical and geometric interpretations of the second and third derivatives.
 40. Calculate second and higher-order derivatives of appropriate functions.
 41. If $s = f(t)$ gives the position of a body moving in a single direction a function of time, find and interpret the velocity and acceleration at a specified instant.
 42. State the definition for the number e as a limit at infinity.
 43. State the limit condition applied to determine the number e and the resulting rule for the derivative of $y = e^x$.
 44. Using implicit differentiation and appropriate exponential and logarithmic identities, find the derivatives of logarithmic functions.
 45. State and apply the general rules for the derivative of a constant non-integer power function.
 46. Using implicit differentiation and appropriate trigonometric identities, find the derivatives of the inverse trigonometric functions.
 47. *State the definition of the hyperbolic sine and cosine in terms of exponentials, and of the hyperbolic tangent, secant, cosecant and cotangent in terms of the hyperbolic sine and cosine.*
 48. *Sketch the graphs of the hyperbolic sine, cosine and tangent.*
 49. *Use given hyperbolic identities either to simplify expressions involving hyperbolic functions, or to calculate the value of any given hyperbolic function given the value of any other.*
 50. *Calculate the derivatives of functions expressed in terms of hyperbolic functions.*
 51. *State the definitions of the six inverse hyperbolic functions.*
 52. *Using appropriate tables, calculate the derivatives of functions containing inverse hyperbolic functions.*
- Note: Items 47 through 52 are covered second semester, only when time permits.*
53. Set up and solve related rates problems.
 54. State the definition of the linear approximation of f at a and of the linearization of f at a .

55. Estimate the change (dy) produced in a function $y = f(x)$ when x changes by a small amount dx .
56. Given an appropriate function $y = f(x)$, find the differential dy .
57. State the definitions of the terms local (relative) maximum, local (relative) minimum, absolute (global) maximum, absolute (global) minimum and extreme value.
58. State under what conditions a function f may have a local extrema at a point.
59. Given an appropriate function f , find the critical numbers (points) of f and correctly categorize each point in terms of the different possible types of extrema.
60. Given an appropriate function f , use the first derivative to determine the values where f is increasing and where it is decreasing.
61. Given an appropriate function f , use the First Derivative Test to determine whether a critical number represents a local minimum, local maximum or neither.
62. State the definitions of concave up, concave down and inflection point, and interpret each graphically.
63. Determine the concavity of a function f at a given point, based on the sign of its second derivative.
64. Given an appropriate function, use the Second Derivative Test to determine whether a critical number represents a local minimum or local maximum. Describe conditions and identify specific instances in which the test may be inconclusive.
65. Given a function, identify symmetries, intercepts and asymptotes, if any, and find the intervals for which the curve is increasing, decreasing, concave upward, and concave downward. Using this information, sketch the curve, properly identifying the points of inflection and the points where the function has local maximum and local minimum values.
66. Know and be able to use L'Hôpital's Rule.
67. Use appropriate algebraic manipulation coupled with L'Hôpital's rule to find limits of indeterminate forms.
68. Set up and solve, using relevant calculus methods where appropriate, optimization problems.
69. State the definition of antiderivative of f .
70. State/find by sight the antiderivatives for x of simple functions.
71. State and apply the rules for the general antiderivatives of constants times functions and sums or differences of functions.
72. Find the general antiderivative of any given polynomial or general sum of powers of x .
73. Given the acceleration, initial velocity, and initial position, find the position at any time t of a body moving in a straight line (rectilinear motion).
74. Estimate the area under a curve by summing the areas of a finite number of approximating rectangles. Describe how the value of the estimate may change depending on how the rectangles are constructed. Estimate the values of other, similar quantities, e.g. distances traveled, volumes of solids, etc., by similar approximations.
75. Interpret and utilize the sigma notation to express or write out sums.
76. State the definition of a Riemann Sum of a function and the definite integral of the same function, and the relationship between them.
77. State and apply rules for the sums of constants, constant multiples, sums and differences, together with formulas for summing consecutive integers, their squares, or their cubes to determine (if possible) the value of a Riemann sum expressed in sigma notation.

78. Convert the limit of a Riemann sum over an interval to the corresponding definite integral.
79. State and apply rules for the definite integrals of constants, constant multiples of known functions, and sums and differences of known functions, for the sums of integrals of the same function over adjacent domains, etc., and the comparison properties of definite integrals.
80. State the Fundamental Theorem of Calculus.
81. Use the Fundamental Theorem of Calculus, along with relevant rules for the definite integral, to calculate, from memory, definite integrals containing sums of (positive or negative) powers of x , or sums of terms containing $\sin(x)$, $\cos(x)$, and e^x .
82. Use the Fundamental Theorem of Calculus to calculate derivatives.
83. State the definition of the indefinite integral of f , and its relationship to the definite integral of f .
84. Find the (net) area between the horizontal axis and the graph of an appropriate continuous function with both positive and negative values on a given interval.
85. Recognize and use the Substitution Rule to find Integrals.
86. Find the area bounded by two given continuous curves $f(x)$ and $g(x)$ over an interval, including calculating the endpoints, if required.
87. Find the area of the region bounded on the left by the continuous curve $x = g(y)$ and on the right by the continuous curve $x = f(y)$ as y varies from $y = a$ to $y = b$. (The calculation of a and b may be required.)
88. Set up finite sums to approximate the volumes of appropriate solids of rotation.
89. Set up finite sums to approximate the work done in some appropriate process. Convert the limiting values of such sums to definite integrals, and evaluate those integrals to compute the actual work.
90. Calculate the average value of an appropriate continuous function over a specified interval, and interpret the result graphically.
91. State and apply to appropriate integrals the formula for integration by parts, including determining the specific functions needed to partition the integrand, and evaluating the resulting integrals.
92. Use appropriate identities to evaluate or simplify integrals involving trigonometric functions.
93. Use trigonometric substitution to evaluate or simplify appropriate integrals.
94. Use long division and partial fractions to decompose an appropriate rational function into a polynomial plus a sum of reciprocal powers of linear and irreducible quadratic terms. Use that result to find the integral of the original rational function.
95. Find indefinite integrals using appropriate integral tables.
96. State the definitions of Improper Integrals.
97. Determine, where possible, by direct computation whether a given improper integral converges or diverges. If convergent, evaluate it.
98. State and use Comparison Theorems, if applicable, to determine the convergence or divergence of improper integrals.
99. State the definition of a sequence, and, given an expression for a sequence, write explicitly the first few items of that sequence.

100. State what is meant by convergence or divergence of a sequence. Given an appropriate sequence, determine whether it converges or diverges, and, where possible, use standard calculus methods to determine the limiting value.
101. State the definitions of a infinite series, and of convergence or divergence of a series.
102. For a given series, correctly distinguish between the sequence of terms and the sequence of partial sums, and write, explicitly, the values of first few terms and the first few partial sums.
103. State the definition of a geometric series, and the specific conditions under which such a series either converges or diverges. Determine whether or not a given geometric series, or series which can be readily transformed into one, converges.
104. Describe generally the purpose of convergence tests for infinite series, why they are necessary, and their limitations.
105. State under what conditions the convergence tests, i.e. the integral test, the nth term test, the comparison test, the limit comparison test, the absolute convergence test, the alternating series test, and the ratio test may and may not be legitimately applied, what results for each test conclusively indicate either convergence or divergence, and what results must be taken as inconclusive.
106. State the difference between absolute and conditional convergence, and for what kinds of series the distinction becomes important. Use standard convergence tests to determine whether a given series is absolutely convergent, conditionally convergent, or divergent.
107. State the definition of a power series. Given a power series, write the first few terms f the series explicitly, and find its interval of convergence.
108. Given a power for a function $f(x)$, find the power series for f .
109. *State the definitions of a Taylor Series and a Maclaurin series, and the differences between them. Given an appropriate function, find the first several terms of either its Taylor Series about a given point, or its Maclaurin series.*
110. *State the Taylor and Maclaurin series for standard functions such as e , $\sin(x)$, etc.*
111. *Use such standard series to simplify or evaluate other appropriate given power series.*
112. *State and apply Taylors Inequality for the remainder to both bound and estimate the error resulting from approximating a given function by a specified number of terms of its Taylor series.*
113. *Write the binomial series for an appropriate given expression and determine where it converges.*

Note: Items 109 through 113 are covered second semester, only as time permits.